# Gauge Theory Without Ghosts

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## Abstract

A quantum effective action for gauge field theories is constructed that is gauge invariant and independent of the choice of gauge breaking terms in the functional integral that defines it. The loop expansion of this effective action leads to new Feynman rules, involving new vertices but without diagrams containing ghost lines. The new rules are given in full for pure Yang-Mills theory, and renormalization procedures are sketched. No BRST arguments are needed. With the new rules the $\beta$ function becomes ghost independent to all orders. Implications for a stably-based ghost-free attack on the back-reaction problem in quantum gravity are briefly discussed.

PACS numbers: 11.15.-q, 04.60.+n, 11.10.Gh

The purpose of this Letter is to show that by using the geometry of the space-of-histories $\Phi$ of a gauge field in a judicious way one can develop a renormalizable perturbation theory in which ghosts play no role.

$\Phi$ is a principal fibre bundle having for its typical fibre the gauge group $\mathcal{G}$. Real physics takes place in the base space $\Phi/\mathcal{G}$. Since $\mathcal{G}$ is a group manifold it admits a group invariant Riemannian metric. This metric can be extended in an infinity of ways to an invariant metric $\gamma$ on $\Phi$, but it turns out that if the extended metric is required to be *ultralocal* then, up to a scale factor, it is unique in the case of Yang-Mills fields and belongs to a one-parameter family in the case of gravity.

The classical action $S$ is invariant under gauge transformations, which have the infinitesimal form

$$\delta\varphi^i = Q^i{}_\alpha \delta\xi^\alpha, \tag{1}$$

the $\varphi^i$ being the fields (which are to be viewed as coordinates in $\Phi$) and the $\delta\xi^\alpha$ being infinitesimal gauge parameters. Here all indices play a double role, specifying discrete labels as well as points of spacetime, which means that summations over repeated indices include integrations over spacetime.

Group invariance of $\gamma$ is the statement

$$\pounds_{\mathbf{Q}_\alpha}\boldsymbol{\gamma} = 0, \tag{2}$$

where the $\mathbf{Q}_\alpha$ are vector fields on $\Phi$ having the components $Q^i_\alpha$. The $\mathbf{Q}_\alpha$ are *Killing vector fields* for $\boldsymbol{\gamma}$ and *vertical vector fields* for the principal bundle $\Phi$. The $\mathbf{Q}_\alpha$ and $\gamma$ together define a unique connection 1-form on $\Phi$:

$$\boldsymbol{\omega}^\alpha = \mathfrak{G}^{\alpha\beta}\mathbf{Q}_\beta \cdot \boldsymbol{\gamma} \tag{3}$$

where $\mathfrak{G}^{\alpha\beta}$ is the Green's function, appropriate to the boundary conditions at hand in a given problem, of the operator

$$\mathfrak{F}_{\alpha\beta} = -\mathbf{Q}_\alpha \cdot \boldsymbol{\gamma} \cdot \mathbf{Q}_\beta. \tag{4}$$

It is easy to see that $\boldsymbol{\omega}^\alpha \cdot \mathbf{Q}_\beta = \delta^\alpha{}_\beta$ and that horizontal vectors are those that are perpendicular to the fibres (under the metric $\boldsymbol{\gamma}$). A horizontal vector may be obtained from any vector by application of the *horizontal projection operator*:

$$\Pi^i{}_j = \delta^i{}_j - Q^i{}_\alpha \omega^\alpha{}_j. \tag{5}$$

The trick for avoiding ghosts is to make use of the following connection on the frame bundle $F\Phi$:

$$\begin{aligned}\Gamma^i{}_{jk} \;=\; & \Gamma_\gamma{}^i{}_{jk} - Q^i{}_{\alpha\cdot j}\omega^\alpha{}_k - Q^i{}_{\alpha\cdot k}\omega^\alpha{}_j \\ & + \frac{1}{2}\omega^\alpha{}_j Q^i{}_{\alpha\cdot l} Q^l{}_\beta \omega^\beta{}_k + \frac{1}{2}\omega^\alpha{}_k Q^i{}_{\alpha\cdot l} Q^l{}_\beta \omega^\beta{}_j.\end{aligned} \tag{6}$$

Here $\Gamma_\gamma$ is the Riemannian connection associated with $\gamma$ and the dots denote covariant differentiation based on it. The connection (6), which was first introduced by G. A. Vilkovisky [1], has the following remarkable properties:

1. Let $\lambda$ be a geodesic based on it. If the tangent vector to $\lambda$ is horizontal at one point along $\lambda$ then (a) it is horizontal everywhere along $\lambda$, (b) $\lambda$ is also a geodesic based on $\Gamma_\gamma$, and (c) $\lambda$ is the horizontal lift of a Riemannian geodesic in $\Phi/\mathcal{G}$ based on the natural projection of $\gamma$ down to $\Phi/\mathcal{G}$ (which exists because of the group invariance of $\gamma$).

2. Alternatively, if $\lambda$ is tangent to a fibre at one point then it lies in that fibre.

3. For all $\alpha$

$$Q^i{}_{\alpha;j} = \frac{1}{2} Q^i{}_\gamma c^\gamma{}_{\alpha\beta} \omega^\beta{}_j, \tag{7}$$

where the semicolon denotes covariant functional differentiation based on Vilkovisky's connection and the $c^\gamma{}_{\alpha\beta}$ are the structure constants of $\mathcal{G}$.
4. If $A$ is a gauge invariant functional (e.g., $A = S$) then for all $n$

$$A_{;(i_1 \ldots i_n)} = A_{\cdot(j_1 \ldots j_n)} \Pi^{j_1}{}_{i_1} \ldots \Pi^{j_n}{}_{i_n}, \tag{8}$$

where the parentheses indicate that a symmetrization of the indices they embrace is to be performed.

5. If one introduces a fibre-adapted coordinate system $(I^A, K^\alpha)$, where the $I^A$ label the fibres (and are gauge invariant) and the $K^\alpha$ label the points in each fibre, then the components of the *nth* covariant functional derivative of any gauge invariant quantity vanishes unless all indices are capital Latin.

In practice the $I^A$ are purely conceptual, but the $K^\alpha$ need to be explicitly chosen. One must single out a *base point* $\varphi_*$ in $\Phi$ and choose the $K$'$s$ so that the matrix

$$\hat{\mathfrak{F}}^\alpha{}_\beta = \mathbf{Q}_\beta K^\alpha \tag{9}$$

is a nonsingular differential (or pseudodifferential) operator at and in a neighborhood of $\varphi_*$. Typical convenient choices for $\varphi_*$ are $A^\alpha{}_\mu(x)_* = 0$ in Yang-Mills theory and $g_{\mu\nu}(x)_* = \eta_{\mu\nu}$ (Minkowski metric) in gravity theory. It turns out to be extremely useful to use the base point also for another purpose.

Let $\varphi$ be an arbitrary point of $\Phi$ and $\lambda$ a geodesic connecting it to the base point. Let $s$ and $s_*$ be the values, at $\varphi$ and $\varphi_*$ respectively, of an affine parameter along $\lambda$. Define

$$\boldsymbol{\phi} = (s - s_*) \left( \frac{\partial}{\partial s} \right)_{\lambda(s_*)}. \tag{10}$$

$\boldsymbol{\phi}$ is a vector at $\varphi_*$, invariant under rescaling of s. Its components $\phi^a$ in an arbitrary frame at $\varphi_*$ may be called *Gaussian normal fields*. Vilkovisky [1] has shown how the $\phi^a$ can be used to carry out covariant functional Taylor expansions about $\varphi_*$. Such expansions are important in the theory of the quantum effective action by virtue of the fact that, in a fibre-adapted coordinate frame, $\phi^A$ depends only on the $I_*^A$ and $I^A$.

One begins by writing

$$e^{i\Gamma[I_*,\bar{I}]} = N\int e^{iS[I_*,I]+i\Gamma_1[I_*,\bar{I}](\bar{I}-I)}\mu[I_*,I][dI]. \tag{11}$$

where $\mu$ is an appropriate measure functional and the quantum effective action $\Gamma$ is understood to be obtainable (in principle) by an iterative procedure based on computing a loop expansion for the functional integral. Here the $I^A$ are assumed to be already Gaussian normal (i.e., they are the $\phi^A$) so that the difference $\bar{I}-I$ is a vector at $I_*$ and makes good geometrical sense when contracted with the functional derivative (gradient) $\Gamma_1$ of $\Gamma$ with respect to $\bar{I}$.

Although eq. (11), which recognizes that real physics takes place in $\Phi/\mathcal{G}$, is a reasonable starting point, it is of formal validity only because the $I^A$ are purely conceptual. For explicit calculations one must pass from $\Phi/\mathcal{G}$ to $\Phi$ by introducing variables $K^\alpha$. These too may be chosen Gaussian normal, in which case both the $I^A$ and the $K^\alpha$ are necessarily linearly related to the $\phi^a$ in an arbitrary frame at $\varphi_*$:

$$I^A = P^A{}_a[\varphi_*]\phi^a, \qquad K^\alpha = P^\alpha{}_a[\varphi_*]\phi^a. \tag{12}$$

($\varphi_*$ can now be any point in the fibre over $I_*$.) Equation (11) can then be rewritten with a Gaussian gauge breaking term thrown in:

$$\begin{aligned} e^{i\Gamma[I_*,\bar{I}]} \;=\; & N\int[dI]\int[dK]e^{i(S[I_*,I]+\frac{1}{2}\kappa_{\alpha\beta}[\varphi_*]K^\alpha K^\beta)+i\Gamma_1[I_*,\bar{I}](\bar{I}-I)} \\ & \times(\det\kappa[\varphi_*])^{\frac{1}{2}}\mu[I_*,I]. \end{aligned} \tag{13}$$

Since $\phi^A$ $(=I^A)$ depends only on the $I_*^A$ and $I^A$ one can immediately transform to Gaussian normal fields in an arbitrary frame, noting, by virtue of (12), that the Jacobian $J[\varphi_*]$ for the transformation is a *constant*. We shall abuse notation slightly by writing $\Gamma[I_*,\bar{I}] = \Gamma[\varphi_*,\bar{\phi}]$ where the $\bar{\phi}^a$ are such that $\bar{I}^A = P^A{}_a[\varphi_*]\bar{\phi}^a$. We stress that, despite the presence of the gauge breaking term, $\Gamma[\varphi_*,\bar{\phi}]$ is still gauge invariant and independent of the choice of $P$'$s$ and $\kappa$'$s$, provided the $\bar{\phi}^a$ are held at values such that $\bar{K}^\alpha = P^\alpha{}_a[\varphi_*]\bar{\phi}^a$ vanishes. However, if the $\bar{\phi}^a$ are allowed to run free then $\Gamma$ suffers merely the simple modification $\Gamma\to\hat{\Gamma}$ where

$$\hat{\Gamma}[\varphi_*,\bar{\phi}] = \Gamma[\varphi_*,\bar{\phi}] + \frac{1}{2}\kappa_{\alpha\beta}[\varphi_*]P^\alpha{}_a[\varphi_*]P^\beta{}_b[\varphi_*]\bar{\phi}^a\bar{\phi}^b. \tag{14}$$

Hence finally

$$e^{i\hat{\Gamma}[\varphi_*,\bar{\phi}]} \;=\; N\int e^{i(S[\varphi_*,\phi]+\frac{1}{2}\kappa_{\alpha\beta}[\varphi_*]P^{\alpha}{}_{a}[\varphi_*]P^{\beta}{}_{b}[\varphi_*]\phi^a\phi^b)+i\hat{\Gamma}_1[\varphi_*,\bar{\phi}](\bar{\phi}-\phi)} \times(\mathrm{det}\kappa[\varphi_*])^{\frac{1}{2}}\mathrm{J}[\varphi_*]\mu[\varphi_*,\phi][\mathrm{d}\phi]. \tag{15}$$

Now let $\hat{\mathfrak{G}}^{\alpha}{}_{\beta}$ be the Green's function of the operator $\hat{\mathfrak{F}}^{\alpha}{}_{\beta}$ of eq. (9). It is easy to show that $J\mathrm{det}\hat{\mathfrak{G}}$ is independent of the choice of the $K$'s (i.e., of the $P^{\alpha}{}_{a}$). Hence, as far as assuring the $P$-independence of $\Gamma$ is concerned, $J$ may be replaced by $(\mathrm{det}\hat{\mathfrak{G}})^{-1}$ in the functional integral. This is the well known ghost determinant. It is not difficult to obtain a covariant functional Taylor expansion of $ln\mathrm{det}\hat{\mathfrak{G}}$, and to use it to compute vertices for the interaction of ghosts with the fields $\phi^a$. These vertices turn out to have such a form as to conspire to cause every Feynman graph containing a ghost line to vanish. It is clear, from the constancy of $J$, that the ghost can play no real diagrammatic role. It has become, as it were, a ghost of itself.

In what follows we shall drop $\mathrm{det}\kappa$, $J$ and the measure from the integrand of eq. (15). (It is shown in reference [2] that the chief role of the measure is to justify throwing away nonvanishing contributions from arcs at infinity in the Wick rotation procedure.) The loop perturbation series is then obtained by expanding the integrand about $\bar{\phi}$, writing

$$\phi = \bar{\phi} + \chi \tag{16}$$

and using

$$S[\varphi_*,\bar{\phi}+\chi] \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}S_{,a_1\ldots a_n}[\varphi_*,\bar{\phi}]\chi^{a_1}\ldots\chi^{a_n}, \tag{17}$$

where the ordinary derivatives of $S[\varphi_*,\bar{\phi}]$ with respect to the $\bar{\phi}^a$ are given by

$$S_{,a_1\ldots a_n}[\varphi_*,\bar{\phi}] \;=\; \sum_{m=0}^{\infty}\frac{1}{m!}S_{;(a_1\ldots a_n b_1\ldots b_m)}[\varphi_*]\bar{\phi}^{b_1}\ldots\bar{\phi}^{b_m}. \tag{18}$$

The loop graphs themselves are embodied in the functional

$$\Sigma[\varphi_*,\bar{\phi}] \;=\; \Gamma[\varphi_*,\bar{\phi}] - S[\varphi_*,\bar{\phi}] \tag{19}$$

which is generated by the functional integral equation (alternative to (15))

$$e^{i\Sigma[\varphi_*,\bar{\phi}]} = N\int e^{i(-\Sigma_1[\varphi_*,\bar{\phi}]\chi+\frac{1}{2}F[\varphi_*,\bar{\phi}]\chi\chi+\frac{1}{6}S_3[\varphi_*,\bar{\phi}]\chi\chi\chi+\ldots)}[d\chi] \tag{20}$$

where

$$F[\varphi_*,\bar{\phi}] \;=\; S_2[\varphi_*,\bar{\phi}] + P^{tr}[\varphi_*]\kappa[\varphi_*]P[\varphi_*]. \tag{21}$$

Amplitudes for physical processes that take place in the background $\varphi_*$ are obtained by attaching external lines to the graphs and setting $\bar{\phi}=0$. The vertex functions (18) then reduce (see eq. (8)) to

$$S_{,a_1\ldots a_n}[\varphi_*,0] = S_{\cdot(b_1\ldots b_n)}[\varphi_*]\Pi^{b_1}{}_{a_1}[\varphi_*]\ldots\Pi^{b_n}{}_{a_n}[\varphi_*], \tag{22}$$

and these are easily calculated. The secret of ghost-free gauge theory is seen to be very simple: Replace all traditional vertex functions by (22) and throw away the ghost diagrams.

The new rules are particularly simple in the case of Yang-Mills theory. The classical action, in a Minkowski spacetime of $N$ dimensions, is

$$S = -\frac{1}{4}\int \gamma_{\alpha\beta} F^{\alpha}{}_{\mu\nu} F^{\beta\mu\nu} d^N x, \tag{23}$$

where $\gamma_{\alpha\beta}$ is the Cartan-Killing metric of the associated Lie algebra (assumed simple and non-Abelian) and

$$F^{\alpha}{}_{\mu\nu} = \partial_\mu A^{\alpha}{}_{\nu} - \partial_\nu A^{\alpha}{}_{\mu} + g_0 f^{\alpha}{}_{\beta\gamma} A^{\beta}{}_{\mu} A^{\gamma}{}_{\nu}, \tag{24}$$

$g_0$ being the bare coupling constant and $f^{\alpha}{}_{\beta\gamma}$ the structure constants of the algebra, related to $\gamma_{\alpha\beta}$ by

$$f^{\gamma}{}_{\alpha\delta} f^{\delta}{}_{\beta\gamma} = -\lambda \gamma_{\alpha\beta} \tag{25}$$

for some positive constant $\lambda$ that depends on the scale choice of the algebra basis. The gauge transformation law (1) takes the form

$$\delta A^{\alpha}{}_{\mu} = \int Q^{\alpha}{}_{\mu\beta'} \delta\xi^{\beta'} d^N x' \tag{26}$$

where

$$Q^{\alpha}{}_{\mu\beta'} = (-\delta^{\alpha}{}_{\beta}\partial_\mu + g_0 f^{\alpha}{}_{\beta\gamma} A^{\gamma}{}_{\mu})\delta(x, x'), \tag{27}$$

and the structure constants of $\mathcal{G}$ are

$$c^{\alpha}{}_{\beta'\gamma''} = g_0 f^{\alpha}{}_{\beta\gamma} \delta(x, x')\delta(x, x''). \tag{28}$$

The unique ultralocal invariant metric is

$$\gamma_{\alpha}{}^{\mu}{}_{\beta'}{}^{\nu'} = \gamma_{\alpha\beta}\, \eta^{\mu\nu} \delta(x, x'), \tag{29}$$

which, being constant, is flat. The Riemannian connection components vanish in the coordinates $A^{\alpha}{}_{\mu}(x)$, and the dots in eqs. (6), (8) and (22) denote ordinary functional derivatives. This means that there are only two distinct vertex functions, $S_3$ and $S_4$, just as in the traditional formalism.

The $P^{\alpha}{}_{a}$ and $\kappa_{\alpha\beta}$ for Yang-Mills fields are conveniently chosen to be

$$P^{\alpha}{}_{\beta'}{}^{\mu'} = -\delta^{\alpha}{}_{\beta}\, \eta^{\mu\nu} \partial_\nu \delta(x, x'), \tag{30}$$

$$\kappa_{\alpha\beta'} = -\gamma_{\alpha\beta}\, \delta(x, x'). \tag{31}$$

When $A^{\alpha}{}_{\mu *} = 0$ and $\bar{\phi} = 0$ the operator (21) then takes the simple form

$$F_{\alpha}{}^{\mu}{}_{\beta'}{}^{\nu'} \to -\gamma_{\alpha\beta}\, \eta^{\mu\nu} p^2, \tag{32}$$

where "$\to$" means "pass to the Fourier transform." Moreover, the horizontal projection operator (5) becomes

$$\Pi^{\alpha}{}_{\mu\beta'}{}^{\nu'} \to \delta^{\alpha}{}_{\beta}(\delta_{\mu}{}^{\nu} - p_{\mu}p^{\nu}/p^2). \tag{33}$$

It is easy to see that one may restate the calculational rules for Yang-Mills theory as follows:

1. Retain only those traditional graphs that contain no ghost lines and, in these, use the traditional vertices.
2. For the internal lines use the Green's function in the Landau gauge.
3. Apply the projection operator (33) to all external prongs.

Renormalization in the ghost-free formalism, although technically requiring as much work as in the traditional formalism, is conceptually simpler. There are only two independent renormalization constants instead of three, a wave function renormalization constant $Z$ and a constant $Y$ that renormalizes the three-pronged vertex. The constant, call it $X$, that renormalizes the four-pronged vertex is fixed by gauge invariance to be $X = Z^{-1}Y^2$. The renormalized fields $A_R{}^{\alpha}{}_{\mu}$ and renormalized coupling constant $g$ are defined by

$$A^{\alpha}{}_{\mu} = Z^{1/2} A_R{}^{\alpha}{}_{\mu}, \qquad g_0 = \mu^{2-N/2} Z^{-3/2} Y g, \tag{34}$$

where $\mu$ is the usual auxiliary mass. Because the Gaussian normal fields are nonlocally related to the $A_R{}^{\alpha}{}_{\mu}$, it turns out that there is an additional *nonlocal* gauge invariant term, of the form

$$\begin{aligned} \Delta S \;=\; & \frac{\Xi}{24} tr(f_{\alpha} f_{\beta} f_{\gamma} f_{\delta})(\eta^{\mu\nu}\eta^{\sigma\tau} + \eta^{\mu\sigma}\eta^{\nu\tau} + \eta^{\mu\tau}\eta^{\nu\sigma}) \\ & \times \int d^N x \int d^N x' \int d^N x'' \int d^N x''' \int d^N x'''' \Pi^{\alpha}{}_{\mu\bar{\alpha}'}{}^{\bar{\mu}'} \Pi^{\beta}{}_{\nu\bar{\beta}''}{}^{\bar{\nu}''} \\ & \times \Pi^{\gamma}{}_{\sigma\bar{\gamma}'''}{}^{\bar{\sigma}'''} \Pi^{\delta}{}_{\tau\bar{\delta}''''}{}^{\bar{\tau}''''} A_R{}^{\bar{\alpha}'}{}_{\bar{\mu}'} A_R{}^{\bar{\beta}''}{}_{\bar{\nu}''} A_R{}^{\bar{\gamma}'''}{}_{\bar{\sigma}'''} A_R{}^{\bar{\delta}''''}{}_{\bar{\tau}''''}, \end{aligned} \tag{35}$$

($f_{\alpha} = (f^{\beta}{}_{\alpha\gamma})$) that must be added to the classical action, whose only role is to mop up some residues coming from the divergent parts of diagrams having four external prongs. This term makes no contribution to the particle content of the theory, plays no role in determining the $\beta$ function, and, under dimensional regularization with minimal subtraction, does not even have to be computed.

Diagrams in the ghost-free formalism have the same degree of divergence as in the traditional formalism, and counter terms are computed in the usual way. For pure Yang-Mills theory one finds

$$Z = 1 - \frac{25}{6}\frac{\lambda}{(4\pi)^2}\frac{1}{N-4}\, g^2 + O(g^4), \tag{36}$$

$$Y = 1 - \frac{11}{4}\frac{\lambda}{(4\pi)^2}\frac{1}{N-4}\, g^2 + O(g^4), \tag{37}$$

$$X = 1 - \frac{4}{3}\frac{\lambda}{(4\pi)^2}\frac{1}{N-4}\, g^2 + O(g^4), \tag{38}$$

$$\Xi = \frac{1}{6}\frac{\lambda}{(4\pi)^2}\frac{1}{N-4}\, g^2 + O(g^4), \tag{39}$$

$$\beta = \mu\frac{dg}{d\mu} = -\frac{7}{2}\frac{\lambda}{(4\pi)^2}\, g^3 + O(g^5). \tag{40}$$

Because the Gaussian normal fields are nonlinearly related to the ${A_R}^\alpha{}_\mu$ the relation between the renormalized coupling constant $g$ and that of the traditional theory, call it $\hat{g}$, is also nonlinear:

$$g = \sqrt{22/21}\, \hat{g} + O(\hat{g}^3). \tag{41}$$

This yields the more familiar $\beta$ function

$$\hat{\beta} = \mu\frac{d\hat{g}}{d\mu} = -\frac{11}{3}\frac{\lambda}{(4\pi)^2}\, \hat{g}^3 + O(\hat{g}^5). \tag{42}$$

The Gaussian normal fields to which one is referring here are simply the Gaussian normal fields based on $A^\alpha{}_{\mu *} = 0$ and viewed in the coordinate system defined by the ${A_R}^\alpha{}_\mu$. It is natural to denote them by ${\phi_R}^\alpha{}_\mu$. The obstacle that for years has prevented theorists from looking at Yang-Mills theory in terms of the ${\phi_R}^\alpha{}_\mu$ is the fact that the ${\phi_R}^\alpha{}_\mu$ are nonlocally related to the ${A_R}^\alpha{}_\mu$. But the nonlocality in the neighborhood of $\bar{\phi}_R = 0$ is effectively just that produced by the projection operator (33) and in the end presents no problem at all.

The ultimate importance of the ghost-free formalism lies not in its demonstration that one can actually get along without ghosts (and the whole paraphernalia of BRST as well!), thus remaining close to the spirit of the classical theory from which one starts. What is important is the fact that one obtains a quantum effective action $\Gamma$ that is independent of the choice of gauge breaking terms and of the ghost propagator (although not independent of the choice of the base point $\varphi_*$). This has implications for attempts to tackle not only problems

with “in-out” boundary conditions, which constitute the usual framework for the quantum effective action, but also problems involving “in-in” boundary conditions, in which one tries to get information about expectation values. By using some variant of the well-known “two-time formalism”, together with Vilkovisky’s ideas, one should be able to construct an “in-in” effective action that is ghost and gauge-breaking independent. This would lead to the possibility of finding gauge-covariant, ghost-independent nonlocal dynamical equations that govern, at least approximately, the evolution of field expectation values.

The most interesting application of such a development would be in quantum gravity. Here, of course, there are formidable obstacles: (a) The algebraic complexity that always arises in gravitational calculations. (b) The fact that the number of basic vertices is no longer finite. (c) The fact that $\gamma$ is no longer flat. (d) The fact that gravity is not perturbatively renormalizable. Nevertheless it would be both useful and feasible to examine the “in-out” and “in-in” one-loop quantum gravity effective actions, in ordinary space and in arbitrary backgrounds. This is a minimum requirement, for example, for setting up a stably-based ghost-independent attack on the black-hole back-reaction problem.

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